Morphological Image Processing

Introduction

Morphology: a branch of biology that deals with the form and structure of animals and plants

Morphological image processing is used to extract image components for representation and description of region shape, such as boundaries, skeletons, and the convex hull

Preliminaries (1)

Reflection

The reflection of a set *B*, denoted *B*, is defined as $B = \{w | w = -b, \text{ for } b \in B\}$

Translation

The translation of a set *B* by point $z = (z_1, z_2)$, denoted $(B)_Z$, is defined as

$$(B)_{z} = \{c \mid c = b + z, \text{ for } b \in B\}$$

Example: Reflection and Translation



a b c

FIGURE 9.1 (a) A set, (b) its reflection, and (c) its translation by *z*.



Preliminaries (2)

Structure elements (SE)

Small sets or sub-images used to probe an image under study for properties of interest

Examples: Structuring Elements (1)



FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

Examples: Structuring Elements (2)

Accommodate the entire structuring elements when its origin is on the border of the original set A

Α

At each location of the origin of B, if B is completely contained in A, then the location is a member of the new set, otherwise it is not a member of the new set.

Origin of B visits

every element of A

a b c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion

With *A* and *B* as sets in Z^2 , the erosion of *A* by *B*, denoted $A \ominus B$, defined

$$A \ominus B = \left\{ z \, | \, (B)_z \subseteq A \right\}$$

The set of all points z such that B, translated by z, is contained by A.

$$A \ominus B = \left\{ z \, | \, (B)_z \cap A^c = \emptyset \right\}$$



FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. ^{2/27}The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Example of Erosion (2)



a b c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wirebond mask. (b)-(d) Image eroded using square structuring elements of sizes $11 \times 11, 15 \times 15,$ and 45×45 , respectively. The elements of the SEs were all 1s.

Dilation

With A and B as sets in Z^2 , the dilation of A by B, denoted $A \oplus B$, is defined as

$$\mathbf{A} \oplus \mathbf{B} = \left\{ z \mid \left(\overline{B} \right)_z \cap A \neq \emptyset \right\}$$

The set of all displacements z, the translated B and A overlap by at least one element.

$$A \oplus B = \left\{ z \mid \left[\left(B \right)_z \cap A \right] \subseteq A \right\}$$

Examples of Dilation (1)





 $\hat{B} = B$

a b c d е FIGURE 9.6 (a) Set *A*. (b) Square structuring element (the dot denotes the origin). (c) Dilation of A by *B*, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this

element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference



Examples of Dilation (2)

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0

a c b

FIGURE 9.7 (a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.



Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \ominus B)^c = A^c \oplus B$$

and

$$\left(A \oplus B\right)^c = A^c \ominus \overline{B}$$

Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection

$$\left(A \ominus B\right)^{c} = \left\{ z \mid \left(B\right)_{z} \subseteq A \right\}^{c}$$



Duality

Erosion and dilation are duals of each other with respect to set complementation and reflection

$$(A \oplus B)^{c} = \left\{ z \mid \left(\overline{B} \right)_{Z} \cap A \neq \emptyset \right\}^{c}$$

Opening and Closing

- Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing tends to smooth sections of contours but it generally fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour

Opening and Closing

The opening of set *A* by structuring element *B*, denoted $A \circ B$, is defined as $A \circ B = (A \ominus B) \oplus B$

The closing of set *A* by structuring element *B*, denoted $A \Box B$, is defined as $A \Box B = (A \oplus B) \ominus B$

Opening

The opening of set *A* by structuring element *B*, denoted $A \circ B$, is defined as $A \circ B = \bigcup \{ (B)_Z | (B)_Z \subseteq A \}$

Example: Opening



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Example: Closing



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.



b c d e f g i h **FIGURE 9.10** Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the

structuring element.

а

Duality of Opening and Closing

Opening and closing are duals of each other with respect to set complementation and reflection

$$(A \Box B)^c = (A^c \circ B)$$

$$(A \circ B)^c = (A^c \Box B)$$

The Properties of Opening and Closing

Properties of Opening

(a) $A \circ B$ is a subset (subimage) of A

Properties of Closing



The Hit-or-Miss Transformation





if *B* denotes the set composed of *D* and its background,the match (or set of matches) of *B* in *A*, denoted $A \circledast B$,

$$A^{\circledast}B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

 $B = (B_1, B_2)$ $B_1 : \text{object}$ $B_2 : \text{background}$ $A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$

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e f **FIGURE 9.12** (a) Set A. (b) A window, W, and the local background of D with respect to W, (W - D).(c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by (W - D). (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

a b

c d

Some Basic Morphological Algorithms (1)

Boundary Extraction

The boundary of a set A, can be obtained by first eroding A by B and then performing the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

Example 1



Example 2



a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Some Basic Morphological Algorithms (2)

Hole Filling

A hole may be defined as a background region surrounded by a connected border of foreground pixels.

Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

Some Basic Morphological Algorithms (2)

Hole Filling

1. Forming an array X_0 of 0s (the same size as the array containing A), except the locations in X_0 corresponding to the given point in each hole, which we set to 1.

2.
$$X_k = (X_{k-1} \oplus B) \cap A^c$$
 $k=1,2,3,...$

Stop the iteration if $X_k = X_{k-1}$



a b c d e f g h i

FIGURE 9.15 Hole filling. (a) Set A(shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

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 X_0

 X_1

 X_2



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Some Basic Morphological Algorithms (3)

Extraction of Connected Components

Central to many automated image analysis applications.

Let A be a set containing one or more connected components, and form an array X_0 (of the same size as the array containing A) whose elements are 0s, except at each location known to correspond to a point in each connected component in A, which is set to 1.

Some Basic Morphological Algorithms (3)

Extraction of Connected Components

Central to many automated image analysis applications.

$$X_{k} = (X_{k-1} \oplus B) \cap A$$

B: structuring element

until
$$X_k = X_{k-1}$$



•



 X_6



FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)-(g) Various steps in the iteration of Eq. (9.5-3).

 X_3

 X_2

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